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Poverty traps

Development economics

Lecture 2, 3: Traditional growth models and poverty traps, and the way towards MDGs $% \left({{\mathcal{T}_{\mathrm{s}}}} \right)$

Vojtěch Bartoš

LMU, April 12, 2016

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Economic growth

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Economic growth

- ► Rapid economic development started some 150 years ago.
 - ► 1820-90: Netherlands a major driver of economic growth: annual growth of 0.2%
- Current rates of about 2% enormous growth rates if one takes into account the **exponential growth**. Time to double GDP:

•
$$x(1+r)^t = 2x$$

•
$$t = \frac{\log(2)}{\log(1+r)}$$

• Example: 2% growth \rightarrow **doubling time**: 35 years

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Economic growth (1870 - 1978)

	<i>Per capita</i> GDP (1970 U.S. \$)					
Country	1870	1913		1978		
Australia	1,340	1,941	(1.4)	4,456	(3.3)	
Austria	491	1,059	(1.2)	3,934	(8.0)	
Belgium	939	1,469	(1.6)	4,795	(5.1)	
Canada	619	1,466	(2.4)	5,210	(8.4)	
Denmark	572	1,117	(2.0)	4,173	(7.3)	
Finland	402	749	(1.9)	3,841	(9.6)	
France	627	1,178	(1.9)	4,842	(7.7)	
Germany	535	1,073	(3.7)	4,676	(8.7)	
Italy	556	783	(1.4)	3,108	(5.6)	
Japan	248	470	(1.9)	4,074	(16.4)	
Netherlands	830	1,197	(1.4)	4,388	(5.3)	
Norway	489	854	(1.7)	4,890	(10)	
Sweden	416	998	(2.4)	4,628	(11.1)	
Switzerland	786	1,312	(1.7)	4,487	(5.7)	
United Kingdom	972	1,492	(1.5)	3,796	(3.9)	
United States	774	1,815	(2.3)	5,799	(7.5)	
Simple average	662	1,186	(1.8)	4,444	(6.7)	

Source: Maddison [1979].

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Economic growth

"I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what exactly? If not, what is it about the 'nature of India' that makes it so?" — Robert Lucas

 \blacktriangleright But growth is very unequal and poor countries have to do a lot to catch up \rightarrow

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Economic growth (unequal starting points)

	PPP estim	ates of GNP capita	A 11111001	
	(u.s.	= 100)	annual growth	
Country	1994	1987	1987–94	
Rwanda	1.3	3.8	Ļ	
Ethiopia	1.7	2.0	Ļ	
India	4.9	4.4	1	
Kenya	5.7	5.1	†	
China	9.7	5.8	ŕ	
Sri Lanka	12.2	10.7	↑	
Indonesia	13.9	10.0	↑	
Egypt	14.4	14.4	_	
Russian Federation	17.8	30.6	1.5	
Turkey	18.2	20.9	Ť	
South Africa	19.8	23.9	Ť	
Colombia	20.6	19.0	*	
Brazil	20.9	24.2		
Poland	21.2	21.4	+	
Thailand	26.9	16.4	3e 1 1	
Mexico	27.2	27.8	· · · · ·	
Argentina	33.7	32.1	+	
Korea, Rep	39.9	27.2	Ť	
Greece	42.2	42.1	Ť	
Spain	53.1	42.1 50.2	, Î	
United Kingdom	69.4	50.2	↑	
Canada	77.1	70.2	Ļ	
France	76.0	03.2	Ļ	
Japan	81.7	75.9	1	
Switzerland	97.2	/4.7	↑	
		104.5	↓	

 More recent data in Stata...

Source: World Bank Development Report [1996].

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Economic growth

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- **But**: What causes growth and how to generate it?
- ► Note: Economic growth is the abstention from current consumption (i.e. translates into investment in the (richer) future).

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- **But**: What causes growth and how to generate it?
- ► Note: Economic growth is the abstention from current consumption (i.e. translates into investment in the (richer) future).
- Commodities:
 - Consumption goods
 - Capital goods
 - ▶ (often these cannot be categorised in a single category)

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- But: What causes growth and how to generate it?
- ► Note: Economic growth is the abstention from current consumption (i.e. translates into investment in the (richer) future).
- Commodities:
 - Consumption goods
 - Capital goods
 - ▶ (often these cannot be categorised in a single category)
- Households save (do not spend everything on consumption), savings are invested by firms (to increase capital stocks)

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• Macroeconomic balance: savings = investments: S(t) = I(t)

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- ► Macroeconomic balance: savings = investments:
 S(t) = I(t)
 - ► Q: Examples?

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- ► Macroeconomic balance: savings = investments: S(t) = I(t)
 - ► Q: Examples?
 - Q: Does this necessarily hold at every period t?

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- Macroeconomic balance: savings = investments: S(t) = I(t)
 - ► Q: Examples?
 - Q: Does this necessarily hold at every period t?
- ► Accounting equation: Y(t) = C(t) + S(t), Y(t)... total GNP (not GDP per capita)

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- Macroeconomic balance: savings = investments: S(t) = I(t)
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• So
$$Y(t) = C(t) + I(t)$$
, as $S(t) = I(t)$

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- ► Macroeconomic balance: savings = investments:
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• So
$$Y(t) = C(t) + I(t)$$
, as $S(t) = I(t)$

Investment increases the stock of next period capital goods K(t + 1). In this period the share of δ depreciates:

$$K(t+1) = (1-\delta)K(t) + I(t)$$

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$$K(t+1) = (1-\delta)K(t) + I(t)$$

► To examine growth, we define the following:

► Savings ratio:

$$s=\frac{S(t)}{Y(t)}$$

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$$K(t+1) = (1-\delta)K(t) + I(t)$$

- ► To examine growth, we define the following:
 - ► Savings ratio:

$$s=\frac{S(t)}{Y(t)}$$

 Capital-output ratio – how much capital is needed to produce one unit of output:

$$\theta = \frac{K(t)}{Y(t)}$$

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 - ► Savings ratio:

$$s=\frac{S(t)}{Y(t)}$$

 Capital-output ratio – how much capital is needed to produce one unit of output:

$$\theta = \frac{K(t)}{Y(t)}$$

► From macroeconomic balance we get:

$$K(t+1) = (1-\delta)K(t) + S(t)$$

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$$K(t+1) = (1-\delta)K(t) + S(t)$$

• We know that
$$S(t) = sY(t)$$
 and $K(t) = \theta Y(t)$

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$$K(t+1) = (1-\delta)K(t) + S(t)$$

- We know that S(t) = sY(t) and $K(t) = \theta Y(t)$
- Plug this into the capital stock equation:

$$\theta Y(t+1) = (1-\delta)\theta Y(t) + sY(t)$$

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$$K(t+1) = (1-\delta)K(t) + S(t)$$

- We know that S(t) = sY(t) and $K(t) = \theta Y(t)$
- Plug this into the capital stock equation:

$$\theta Y(t+1) = (1-\delta)\theta Y(t) + sY(t)$$

• Then divide by θ and by Y(t) to get:

$$rac{Y(t+1)}{Y(t)} = rac{Y(t)}{Y(t)}(1-\delta+rac{s}{ heta})$$

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$$\frac{Y(t+1)}{Y(t)} = \frac{Y(t)}{Y(t)}(1-\delta+\frac{s}{\theta})$$

• Subtract
$$\frac{Y(t)}{Y(t)}$$
 to get:

$$\frac{Y(t+1)-Y(t)}{Y(t)}=\frac{s}{\theta}-\delta$$

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$$\frac{Y(t+1)}{Y(t)} = \frac{Y(t)}{Y(t)}(1-\delta+\frac{s}{\theta})$$

• Subtract
$$\frac{Y(t)}{Y(t)}$$
 to get:

$$rac{Y(t+1)-Y(t)}{Y(t)}=rac{s}{ heta}-\delta$$

• And we get the Harrod-Domar equation $(g = \frac{Y(t+1)-Y(t)}{Y(t)})$:

$$\frac{s}{\theta} = g + \delta$$

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Harrod-Domar model and population growth



FIGURE 2. POPULATION AND PER CAPITA GDP OVER THE VERY LONG RUN

Notes: Population and GDP per capita for "the West," defined as the sum of the United States and 12 western European countries. Both series are normalized to take the value 1.0 in the initial year, 1 AD.

Source: Maddison (2008).

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Harrod-Domar model and population growth

$$g=\frac{s}{\theta}-\delta$$

 Population growth requires more capital (i.e. requires higher investment to sustain *per capita* growth)

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Harrod-Domar model and population growth

$$g = \frac{s}{\theta} - \delta$$

- Population growth requires more capital (i.e. requires higher investment to sustain *per capita* growth)
- Population increases at the rate of *n*:

$$P(t+1) = P(t)(1+n)$$

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Harrod-Domar model and population growth

$$g = \frac{s}{\theta} - \delta$$

- Population growth requires more capital (i.e. requires higher investment to sustain *per capita* growth)
- ▶ Population increases at the rate of *n*:

$$P(t+1) = P(t)(1+n)$$

• Let per capita income be: $y(t) = \frac{Y(t)}{P(t)}$

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Harrod-Domar model and population growth

$$g = \frac{s}{\theta} - \delta$$

- Population growth requires more capital (i.e. requires higher investment to sustain *per capita* growth)
- ▶ Population increases at the rate of *n*:

$$P(t+1) = P(t)(1+n)$$

• Let per capita income be: $y(t) = \frac{Y(t)}{P(t)}$

$$heta y(t+1)rac{P(t+1)}{P(t)} = (1-\delta) heta y(t) + sy(t)$$

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Harrod-Domar model and population growth

$$heta y(t+1)rac{P(t+1)}{P(t)} = (1-\delta) heta y(t) + sy(t)$$

• Divide the whole equation by $y(t)\theta$:

$$\frac{y(t+1)}{y(t)}\frac{P(t+1)}{P(t)} = (1-\delta) + \frac{s}{\theta}$$

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Harrod-Domar model and population growth

$$\theta y(t+1) \frac{P(t+1)}{P(t)} = (1-\delta)\theta y(t) + sy(t)$$

• Divide the whole equation by $y(t)\theta$:

$$\frac{y(t+1)}{y(t)}\frac{P(t+1)}{P(t)} = (1-\delta) + \frac{s}{\theta}$$

• Note that
$$\frac{y(t+1)}{y(t)} = \frac{y(t+1)-y(t)+y(t)}{y(t)} = 1 + g_{pc}$$

• Per capita growth rate: $g_{pc} = \frac{y(t+1)-y(t)}{y(t)}$

• **Recall**:
$$\frac{P(t+1)}{P(t)} = (1+n)$$

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Harrod-Domar model and population growth

$$heta y(t+1)rac{P(t+1)}{P(t)} = (1-\delta) heta y(t) + sy(t)$$

• Divide the whole equation by $y(t)\theta$:

$$\frac{y(t+1)}{y(t)}\frac{P(t+1)}{P(t)} = (1-\delta) + \frac{s}{\theta}$$

- ► Note that $\frac{y(t+1)}{y(t)} = \frac{y(t+1)-y(t)+y(t)}{y(t)} = 1 + g_{pc}$
 - Per capita growth rate: $g_{pc} = \frac{y(t+1)-y(t)}{y(t)}$

• **Recall**:
$$\frac{P(t+1)}{P(t)} = (1+n)$$

And we get the per capita Harrod-Domar equation:

$$\frac{s}{\theta} = (1 + g_{pc})(1 + n) - (1 - \delta)$$

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Harrod-Domar model and population growth

$$\frac{s}{\theta} = (1+g_{pc})(1+n) - (1-\delta)$$

► We can disregard the product g_{pc}n, since both are usually very small. Q: When not?

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Harrod-Domar model and population growth

$$\frac{s}{\theta} = (1+g_{pc})(1+n) - (1-\delta)$$

- ► We can disregard the product g_{pc}n, since both are usually very small. Q: When not?
- Then we get the approximate per capita Harrod-Domar equation:

$$\mathsf{g}_{\mathsf{pc}} pprox rac{\mathsf{s}}{ heta} - (\mathsf{n} + \delta)$$

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Sachs (2004): Harrod-Domar evidence

Table 7. Economic Growth Predicted from National Saving, Population Growth, and Capital Consumption, by Developing Region, 1980–2001

Percent^a

Region	Gross national saving as share of gross national income ^a	Growth in populationª	Consumption of fixed capital as share of gross national income ^a	Annual growth in output per capita	
				Predicted	Actual
Tropical sub-Saharan Africa ^b	11.1	2.7	9.9	-2.3	-0.4
South Asia	20.0	2.0	8.7	1.8	3.3
Latin America	18.7	1.8	9.8	1.2	0.4
East Asia and Pacific	35.1	1.3	9.6	7.2	6.4
Middle East and North Africa	23.5	2.4	9.2	2.3	1.0

Source: Authors' calculations using data from World Bank (2003a).

a. Annual average across countries and years, weighted by population.

b. Countries listed in table 2, except Dem. Rep. of Congo and Liberia, for which relevant data are unavailable.
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Figure 6. Growth in Gross National Income by Developing Region, Actual and Predicted, 1980–2001

Predicted (percent a year)



Source: World Bank (2003a) and authors' calculations using model described in the text.

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Beyond Harrod-Domar model

$$g_{pc} \approx rac{s}{ heta} - (n+\delta)$$

Recipes on how to increase growth?

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$$g_{pc} \approx rac{s}{ heta} - (n+\delta)$$

Recipes on how to increase growth?

- 1. Increase the (household) savings rate. How?
- 2. Reduce the capital output ratio (production efficiency). How?
- 3. Reduce the the population growth. How?

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$$g_{pc} \approx rac{s}{ heta} - (n+\delta)$$

- Recipes on how to increase growth?
 - 1. Increase the (household) savings rate. How?
 - 2. Reduce the capital output ratio (production efficiency). How?
 - 3. Reduce the the population growth. How?
- ► All of the above can be *endogenous* (savings, population growth, capital-output or technology).

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Beyond Harrod-Domar model: endogenous savings

 Poverty trap of savings: you cannot start saving unless you reach certain threshold (subsistence level)

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Beyond Harrod-Domar model: endogenous savings

- Poverty trap of savings: you cannot start saving unless you reach certain threshold (subsistence level)
 - ▶ One of reasons for Sachs et al. (2004): MDGs & big push



Source: Do the Rich Save More? Journal of Political Economy, 2004

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Beyond Harrod-Domar model: endogenous savings

- Poverty trap of savings: you cannot start saving unless you reach certain threshold (subsistence level)
 - ▶ One of reasons for Sachs et al. (2004): MDGs & big push



► Note: correlation vs. causation (savings vs. growth)

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Beyond Harrod-Domar model: endogenous population





- Why do poor countries have so different distributions?
- Why do poor countries have such high fertility rates?

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Solow model

- Constant returns to capital?
 - Recall previous lecture and the Lucas Paradox: Capital and labor work together. Capital should be most productive where there is abundance of (cheap) labor.

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- Constant returns to capital?
 - Recall previous lecture and the Lucas Paradox: Capital and labor work together. Capital should be most productive where there is abundance of (cheap) labor.
- Note: capital now transforms to product using a production function in combination with labor. Further we relax the assumption of constant returns of capital. E.g., Cobb-Douglas:

$$Y(t) = A(t)K(t)^{\alpha}P(t)^{1-\alpha}$$

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- Constant returns to capital?
 - Recall previous lecture and the Lucas Paradox: Capital and labor work together. Capital should be most productive where there is abundance of (cheap) labor.
- Note: capital now transforms to product using a production **function** in combination with labor. Further we relax the assumption of constant returns of capital. E.g., Cobb-Douglas:

$$Y(t) = A(t)K(t)^{\alpha}P(t)^{1-\alpha}$$

Recall:

- Technology: $A = \frac{1}{a}$
- Macroeconomic balance: S(t) = I(t)
- Saving rate: $s = \frac{S(t)}{Y(t)}$
- ► Capital accumulation: $K(t+1) = (1 \delta) K(t) + sY(t)$

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$$K(t+1) = (1-\delta)K(t) + sY(t)$$

► Notice: We still assume exogenous s and will assume that population growth is constant (n). Why?

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$$K(t+1) = (1-\delta)K(t) + sY(t)$$

- ► Notice: We still assume exogenous s and will assume that population growth is constant (n). Why?
- ► Rewrite the capital accumulation in per-capita terms again:

$$(1+n)k(t+1) = (1-\delta)k(t) + sy(t)$$

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$$K(t+1) = (1-\delta)K(t) + sY(t)$$

- ► Notice: We still assume exogenous s and will assume that population growth is constant (n). Why?
- ► Rewrite the capital accumulation in per-capita terms again:

$$(1+n)k(t+1) = (1-\delta)k(t) + sy(t)$$

► Production per capita using Y(t) = A(t)K(t)^αP(t)^{1-α} is then:

$$y(t) = A(t)k(t)^{lpha}$$

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Solow model: diminishing returns to capital

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{\alpha}$$

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Solow model: diminishing returns to capital

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{\alpha}$$

• So:
$$f'(k) = \alpha A(t)k^{\alpha-1}$$

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Solow model: diminishing returns to capital

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{\alpha}$$

• So:
$$f'(k) = \alpha A(t)k^{\alpha-1}$$
 and $f''(k) = \alpha(\alpha-1)A(t)k^{\alpha-2}$

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Solow model: diminishing returns to capital

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{lpha}$$

► So:
$$f'(k) = \alpha A(t)k^{\alpha-1}$$
 and $f''(k) = \alpha(\alpha-1)A(t)k^{\alpha-2}$

• Recall: k(t) = K(t)/P(t)

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Solow model: diminishing returns to capital

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{lpha}$$



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Solow model: diminishing returns to capital

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 and $f''(k) = \alpha(\alpha - 1)A(t)k^{\alpha-2}$
► Recall: $k(t) = K(t)/P(t)$



 output-capital ratio falls with labor shortage (diminishing returns)!

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Solow model: dynamics

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{\alpha}$$



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Solow model: dynamics

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{\alpha}$$



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Solow model: steady state

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{\alpha}$$

• Steady state: k^* where k(t) = k(t+1)

$$(1+n-1+\delta)k^* = sA(t)(k^*)^{\alpha}$$

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Solow model: steady state

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{\alpha}$$

• Steady state: k^* where k(t) = k(t+1)

$$(1+n-1+\delta)k^* = sA(t)(k^*)^{\alpha}$$

$$(k^*)^{1-lpha} = \frac{sA(t)}{n+\delta}$$

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Solow model: steady state

$$(1+n)k(t+1) = (1-\delta)k(t) + sA(t)k(t)^{\alpha}$$

• Steady state: k^* where k(t) = k(t+1)

$$(1+n-1+\delta)k^*=sA(t)(k^*)^lpha$$

$$(k^*)^{1-lpha} = rac{sA(t)}{n+\delta}$$

$$k^* = \left(\frac{sA(t)}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

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Mankiw, Romer, and Weil (1992): Solow evidence

ESTIMATION OF THE TEXTBOOK SOLOW MODEL						
Dependent va	Dependent variable: log GDP per working-age person in 19					
	Non-oil	Intermediate				
ons:	98	75				
NT	5.48	5.36				
	(1.59)	(1.55)				
)	1.42	1.31				
	(0.14)	(0.17)				

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ESTIMATION	OF	THE	TEXTBOOK	Solow	MODEL

Dependent variable.	log abi per wor	king-age person in 156	50
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	-1.97	-2.01	-0.76
	(0.56)	(0.53)	(0.84)
\overline{R}^2	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
\overline{R}^2	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
<i>p</i> -value	0.38	0.26	0.79
Implied a	0.60	0.59	0.36
-	(0.02)	(0.02)	(0.15)
	()=)	(),,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	(0120)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the ' 그▶ ≪ @ ▶ ≪ 토▶ ≪ 토▶ – 토 – '오♀? 29/45 period 1960-1985. $(g + \delta)$ is assumed to be 0.05.

- Implicit assumptions: countries in steady state
- We'll calculate this in the tutorial, but: estimated values of α as in Y(t) = $A(t)K(t)^{\alpha}P(t)^{1-\alpha}$ too large.
- Inputing the realistic value of $\alpha = 0.3$ yields R^2 of 0.29 (intermediate sample)

Harrod-Domar model

Solow model

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Poverty traps

Solow model: implications

- Savings no long-term effect on growth of per capita income (long-run growth equal to population growth):
 - ► What about Harrod-Domar model (growth vs. level effects)?

Harrod-Domar model

Solow model

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Solow model: implications

- Savings no long-term effect on growth of per capita income (long-run growth equal to population growth):
 - ► What about Harrod-Domar model (growth vs. level effects)?
- Higher $n \Rightarrow \downarrow k^*$ and \uparrow total output

Harrod-Domar model

Solow model

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Solow model: implications

- Savings no long-term effect on growth of per capita income (long-run growth equal to population growth):
 - ► What about Harrod-Domar model (growth vs. level effects)?
- Higher $n \Rightarrow \downarrow k^*$ and \uparrow total output
- To examine now:
 - 1. Need to study technological progress (A, or $\frac{1}{\theta}$).
 - Hypothesis of international convergence (every country converges to k*, irrespective of the historical starting point)
 - 3. Assumption: marginal product of capital highest where capital least available

Harrod-Domar model

Solow model

Convergence

Poverty traps

Solow model: Technical progress



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Harrod-Domar model

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Solow model: Technical progress



$$k^* = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

 Technology affects the level effects

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Harrod-Domar model

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Solow model: Technical progress



$$k^* = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

- Technology affects the level effects
- Only constant technical progress increases growth persistently

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Harrod-Domar model

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Solow model: Technical progress



$$k^* = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

- Technology affects the level effects
- Only constant technical progress increases growth persistently
- ► Two questions:
 - 1. What is the "technical progress"?
 - 2. Why and when technical progress arises?

Harrod-Domar model

Solow model

Convergence

Poverty traps

Economic growth

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Solow model

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Poverty traps

Solow model: Unconditional convergence

- Do data support the conclusions of the Solow model that in the long term all countries should converge to the same k* and that the richest countries should stop growing (unless persistent differences in technical progress, savings, and population growth)?
 - How to test this empirically?
Harrod-Domar model

Solow model

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Poverty traps

Solow model: Unconditional convergence

- Do data support the conclusions of the Solow model that in the long term all countries should converge to the same k* and that the richest countries should stop growing (unless persistent differences in technical progress, savings, and population growth)?
 - How to test this empirically?
 - $g = \alpha + \beta \log(y_{t0}) + \varepsilon$

Harrod-Domar model

Solow model

Convergence •00000

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Poverty traps

Solow model: Unconditional convergence

- Do data support the conclusions of the Solow model that in the long term all countries should converge to the same k* and that the richest countries should stop growing (unless persistent differences in technical progress, savings, and population growth)?
 - How to test this empirically?
 - $g = \alpha + \beta \log(y_{t0}) + \varepsilon$
 - What would be the Harrod-Domar model prediction?

Harrod-Domar model

Solow model

Convergence

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Solow model: Unconditional convergence

Figure 1: Annual growth rate of GDP per capita between 1870 and 1979 and log GDP per worker in 1870 (16 countries, Baumol, 1986)



Harrod-Domar model

Solow model

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Solow model: Unconditional convergence

Figure 1: Annual growth rate of GDP per capita between 1870 and 1979 and log GDP per worker in 1870 (16 countries, Baumol, 1986)



Selection issue: countries that were rich *ex-post* selected.

Harrod-Domar model

Solow model

Convergence

Poverty traps

Solow model: Unconditional convergence

Figure 1: Annual growth rate of GDP per capita between 1870 and 1979 and log GDP per worker in 1870 (16 countries, Baumol, 1986)



Selection issue: countries that were rich *ex-post* selected.

Figure 2: Annual growth rate of GDP per capita between 1870 and 1979 and log GDP per worker in 1870 (15 + 7 countries, DeLong, 1988)



Harrod-Domar model

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Solow model: Unconditional convergence

Figure 3: Annual growth rate of GDP per capita between 1960 and 2000 and log GDP per worker in 1960 (world)



Source: Acemoglu (2007)

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Solow model

Convergence

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Solow model: Unconditional convergence

► So is Harrod-Domar better than Solow?

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Harrod-Domar model

Solow model

Convergence

Poverty traps

Solow model: Unconditional convergence

- ► So is Harrod-Domar better than Solow?
 - ► Hardly, constant returns to capital unrealistic assumption.

Harrod-Domar model

Solow model

Convergence

Poverty traps

Solow model: Unconditional convergence

- ► So is Harrod-Domar better than Solow?
 - ▶ Hardly, constant returns to capital unrealistic assumption.
- ► But countries can all have different saving rates, levels of technology, or population growth → conditional convergence to different k*

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Harrod-Domar model

Solow model

Convergence 0000●0 Poverty traps

Solow model: Conditional convergence



- AB and A'B' converge to different states due to differences in s and n, but given constant technology lines parallel
 - Q: Why constant technology assumed?
- Q: What can we say about growth of initially poorer and richer countries?

Harrod-Domar model

Solow model

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Solow model: Conditional convergence

Figure 5: Annual growth rate of GDP per capita between 1960 and 2000 and log GDP per worker in 1960 (OECD)



Source: Acemoglu (2007)

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Harrod-Domar model

Solow model

Convergence

Poverty traps

Economic growth

Harrod-Domar model

Solow model

Convergence

Poverty traps

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Harrod-Domar model

Solow model

Convergence

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Returns to capital: poverty trap

- Solow model assumptions for convergence:
 - 1. Savings rate constant for all levels of income: No!

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Solow model

Convergence

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Returns to capital: poverty trap

- Solow model assumptions for convergence:
 - 1. Savings rate constant for all levels of income: No!
 - 2. Population growth constant for all levels of income: No!

Harrod-Domar model

Solow model

Convergence

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Returns to capital: poverty trap

- Solow model assumptions for convergence:
 - 1. Savings rate constant for all levels of income: No!
 - 2. Population growth constant for all levels of income: No!
 - 3. Highest returns to capital for the poorest sf(k)?

Harrod-Domar model

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Returns to capital: poverty trap

- Solow model assumptions for convergence:
 - 1. Savings rate constant for all levels of income: No!
 - 2. Population growth constant for all levels of income: No!
 - 3. Highest returns to capital for the poorest sf(k)?
- What if some threshold level of capital is required for production using more efficient technologies. Why?

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Harrod-Domar model

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Capital threshold in Harod-Domar model

► Recall Harod-Domar:

$$g_{pc} pprox rac{s}{ heta} - (n + \delta)$$

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Harrod-Domar model

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Capital threshold in Harod-Domar model

Recall Harod-Domar:

$$g_{pc} pprox rac{s}{ heta} - (n+\delta)$$

- What if only for certain levels of capital $k > k_T : \frac{s}{\theta} > (n + \delta)$
- Only then g > 0; negative growth for low capital levels poverty trap

Harrod-Domar model

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Capital threshold in Harod-Domar model



Harrod-Domar model

Solow model

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McKenzie and Woodruff (2003): Do Entry Costs Provide an Empirical Basis for Poverty Traps?

	Semiparametric Median Returns by Capital Stock Range			
	\$0-\$200	\$200-\$400	\$400-\$600	\$600-\$1,000
Model 2	22.6	6.5	5.4	4.7
Robustness to low capital stock:				
Dropping bottom 5% of capital	24.1	6.5	5.3	4.8
Dropping bottom 10% of				
capital	23.1	6.4	5.3	4.8
Dropping bottom 25% of				
capital	14.1	6.4	5.4	4.9
Robustness to profit measure:				
Profits = revenues - expenses	18.0	8.4	7.2	6.6
Robustness to accounting system:				
Sample that uses an account-				
ing system	11.7	8.5	6.3	3.9

► Use cross-section of Mexican microenterprises.

Harrod-Domar model

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Robustness to accounting system:				
Sample that uses an account-				
ing system	11.7	8.5	6.3	3.9

► Use cross-section of Mexican microenterprises.Why methodologically problematic?

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De Mel, McKenzie, and Woodruff (2008): Returns to capital in microenterprises

• Q: Are returns to capital so low for the poorest?

De Mel, McKenzie, and Woodruff (2008): Returns to capital in microenterprises

- Q: Are returns to capital so low for the poorest?
- ► Experiment with small firms in Sri Lanka (≈ \$250 non-housing capital)
- Firms divided in three groups (Why?):
 - 1. Received nothing, just observed (control group)
 - 2. Received \$100 (treatment group)
 - 3. Received \$200 (cash or in-kind, randomly)

De Mel, McKenzie, and Woodruff (2008): Returns to capital in microenterprises

- Q: Are returns to capital so low for the poorest?
- ► Experiment with small firms in Sri Lanka (≈ \$250 non-housing capital)
- Firms divided in three groups (Why?):
 - 1. Received nothing, just observed (control group)
 - 2. Received \$100 (treatment group)
 - 3. Received \$200 (cash or in-kind, randomly)
- ► Profits increased by 6% per month (≈ 60% per year) → high returns to capital. Trap?
- Returns were higher for men relative to women. That is a puzzle, since most of microenterprises in developing countries are run by women.

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De Mel, McKenzie, and Woodruff (2008)

$$Y_{it} = \alpha + \sum_{g=1}^{4} \beta_g \operatorname{Treatment}_{git} + \sum_{t=2}^{9} \delta_t + \lambda_i + \varepsilon_{it}$$

Capital Log capital Real Log real Owner Impact of treatment stock stock profits profits hours worked amount on: (1)(2)(3)(4)(5)10,000 LKR in-kind 4.793^{*} 0 40*** 186 0.10 6.06** (2.714)(0.077)(387)(0.089)(2.86)20.000 LKR in-kind 0.71^{***} 13.167*** 1.022^{*} 0.21^{*} -0.57(3.773)(0.169)(592)(0.115)(3.41)10,000 LKR cash 10,781** 0.23^{**} 1,421*** 0.15^{*} 4.52^{*} (5,139)(0.103)(493)(0.080)(2.54)20,000 LKR cash 23.431*** 0 53*** 775^{*} 0.21^{*} 2.37(6.686)(0.111)(643)(0.109)(3.26)385 385 385 385 Number of enterprises 385Number of observations 3,378 3,1553,1553,2483,248

EFFECT OF TREATMENTS ON OUTCOMES

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